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Mathematical Practice One: Make Sense of Problems and Persevere in Solving Them

> Don't be discouraged. It's often the last key in the bunch that opens the lock.
>
> **- Anonymous**

According to the **Common Core State Standards**, mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Educators are faced with the challenge of teaching their students mathematical content, along with the thinking and habits of mind required to "do mathematics." Teachers, by engaging in problems of their own and discussing the mathematical practices with colleagues, analyzing each of the practices, and planning for ways to explicit model the practices, will be poised and ready to do this teaching.

Problem Solving Proficiency

Students who are mathematically proficient with problem solving:

Explain the meaning of the problem and restate it in their own words.
Students engage in close reading of the problem, breaking it down into meaningful parts and restating important information and ideas in their own words. Students are very clear about what the question is asking of them. Some students find the use of highlighters or underlining with their pencil helpful in this step of problem solving. This step in the problem solving process can be challenging for teachers, because ultimately students should be engaged in a productive struggle and the teacher should not "give it away." As long as they are not solving the problem and not taking away from students' ability to reflect and create a plan for solving, it is appropriate for teachers to share information and model reading of the problem.

Researchers James Hiebert and Douglas Grouws suggest three practices:

1. Name mathematical conventions, including symbols (i.e. +, =, x), terminology, and labels, after concepts have been taught and developed.

Mathematically proficient students can:

1. Explain the meaning of the problem and restate it in their own words
2. Find an entry point for the solution
3. Analyze the given information and decide on a plan to solve the problem, or a strategy to use when solving the problem
4. Implement their plan and solve the problem
5. Monitor progress towards their solution and change their plan if necessary
6. Check for accuracy and reasonableness of work and answer

2. If alternative ways to solve the problem do not come from students, teachers should introduce them as "another way" not the "best way."
3. If students don't make explicit connections between mathematical ideas, teachers should help them clarify their thinking or explanations. For example, when a student highlights a strategy that would be helpful in other context, it's important for the teacher to help the other students to see those connections.

Find an entry point for the solution.
Students need experience solving non-routine problems. Non-routine problems have varying degrees of challenge, do not have a predictable and known approach for solving, and can be represented in multiple ways. It is important to use problems with multiple entry points, or in other words, problems that can be tackled in a variety of different ways. Students are finding the problems' entry point when they identify where they want to start and what information will be helpful to them in developing a plan.

Analyze the given information and decide on a plan to solve the problem, or a strategy to use when solving the problem.
When deciding on a plan to solve the problem, students should rely on their prior knowledge and consider similar problems they may have solved in the past. They may also find it helpful to use simpler forms of the original problem, for instance using smaller numbers or a simpler situation. It is important that students construct a plan for solving the problem, rather than just attempting to solve it.

It can be helpful for students to have practice with the first three steps mentioned above, analyzing problems, finding an entry point, and creating a plan, but not necessarily solving the problem. Teachers can develop problems like this by modifying word problems. For instance, rather than asking students for the answer, ask "How could you figure out ______________?" or "What are some ways you could solve this problem?"

Implement their plan and solve the problem.
As students solve problems, it is important that they make their thinking visible and represent their strategies on paper.

Monitor progress towards their solution and change their plan if necessary.
As students solve problems, they should continually ask themselves "Does this make sense?" This involves metacognition, an awareness of how and why they are solving the problem. Metacognition can be learned through explicit classroom instruction (see the **Think Aloud** on the next page). During discussions, teachers can help students develop metacognitive skills by continually having students talk about the problems they are solving, listen to the strategies and thinking of others, and be thinking about alternative ways they can solve problems. Students should also be in tune to when their thinking is appropriate and relevant, and when there are flaws in their thinking that is leading them down the wrong path.

Check for accuracy and reasonableness of work and answer.
If they are able, students should check their work using a different strategy. In addition to communicating their own solutions, when students hear the reasoning and solutions of others, they are continually comparing it to their own methods, finding connections and possible flaws in thinking.

There is nothing more powerful for teaching problem solving and metacognition when problem solving than the **Think Aloud**. During a **Think Aloud,** the teacher makes thought processes visible by talking through them in front of students while providing necessary visual representation. The rationale for using a **Think Aloud** is that both the teacher and students can focus on the content of the problem, while also focusing on specific mathematical practices necessary to solve the problem. In the example provided, students see how to attack a problem, find an entry point, and consider alternate solutions. The **Think Aloud** allows learners to engage by visibly seeing the thinking, then later reflect on that thinking and apply it as they solve problems on their own.

Think Aloud Exemplar

On a SmartBoard, whiteboard, or chart paper, write the problem: Dave has 8 video games. Matt has 4 times as many video games. How many video games does Matt have? (This problem illustrates multiplicative comparison, 4.OA.1, 4.OA.2)

<table>
<tr><th>Teacher's Thinking</th><th>Strategy Modeled</th></tr>
<tr><td>In this problem, we are comparing the number of video games that Dave has to the number of video games that Matt has using the information given. We know that Dave has 8, and Matt has 4 times as many as that.</td><td>Explain the meaning of the problem and restate in their own words.

Begin by trying to understand what the problem is asking.</td></tr>
<tr><td>Would a picture work to help me understand the relationships between Dave's video game and Matt's video games?

How could I represent this as an equation?

If I write an equation, what is known and unknown? What do the 8 and 4 represent?</td><td>Find an entry point for the solution.

Ask yourself questions that lead to an entry point into the problem.</td></tr>
<tr><td>We have worked on multiplication of equal groups, and used strategies like arrays and repeated addition to help us in solving those problems.

I can draw a picture and write an equation. My picture would look like this:

<table>
<tr><th>Dave's Video Games</th><th>Matt's Video Games</th></tr>
<tr><td>XXXX
XXXX</td><td>XXXX XXXX
XXXX XXXX

XXXX XXXX
XXXX XXXX</td></tr>
</table>

My equation would say:

8 x 4 = ___</td><td>Analyze the given information and decide on a plan to solve the problem, or a strategy to use when solving the problem.

Look for connections to similar problems, possibly with smaller numbers.</td></tr>
<tr><td>I am going to solve this problem by using skip counting by multiples of 8.

8, 16, 24, 32. I needed to skip count four times because Matt had four times as many video games as Dave.

I can also show my skip counting in a table.

<table>
<tr><td>Times as many as Dave</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Matt's number of video games</td><td>8</td><td>16</td><td>24</td><td>32</td></tr>
</table></td><td>Implement the plan and solve the problem.

Solve the problem and represent your thinking.</td></tr>
<tr><td>Does this make sense?</td><td>Monitor progress towards the solution and change the plan if necessary.</td></tr>
<tr><td>I remember that we have practiced using arrays to solve multiplication problems. I can create an array to show four times as many as eight, and then count the squares which could represent the number of video games.

<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>16</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>24</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>32</td></tr>
</table>
8 x 4 = 32</td><td>Check for accuracy and reasonableness of work and answer.

Try solving the problem a different way to check your work.</td></tr>
</table>

Students can use their understanding of multiplicative comparison to solve problems with unknown products, unknown group sizes, and unknown numbers of groups. Students will have the foundational understanding necessary to solve other single or multi-step non-routine problems asking them to multiply or divide using multiplicative comparison such as:

- On Monday, the pet store had 24 rabbits for sale. On Tuesday there were 3 times as many rabbits for sale as there were on Friday. How many rabbits did the pet store have for sale on Friday?
- Mr. Smith bicycled 456 miles in June. He bicycled 3 times as many miles in June as he did in May. He bicycled 4 times as many miles in May as he did in April. What was the total number of miles Mr. Smith bicycled in April?

As students are learning to use these practices on their own, teachers are able to discover what is challenging for their students, what comes easily for their students, and the diverse ways their students represent their thinking. It is worthwhile and meaningful work, all intended to prepare our students for college and career, in a world that will require the ability to solve problems, anticipate new problems, and negotiate situations impossible for us to foresee.

Resources and References

"Common Core Standards for Mathematical Practice." Los Altos, CA: Inside Mathematics. Accessed at: www.insidemathematics.org/index.php/commmon-core-math-intro.
These video resources illustrate the **Standards for Mathematical Practice**.

Hiebert, James and Douglas Grouws. "The Effects of Classroom Mathematics Teaching on Students' Learning." ***Second Handbook of Research on Mathematics Teaching and Learning***. Charlotte, NC: Information Age Publishing, 2007. Accessed at: www.carnegiefoundation.org/sites/default/files/Hiebert_Grouws.pdf.

"Mathematics K-5." Georgia Standards. Georgia Department of Education. Accessed at: www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx.

"Mathematical Practices." Carroll County Public Schools. Accessed at: www.carrollk12.org/instruction/instruction/elementary/math/curriculum/common.
This site makes the **Standards for Mathematical Practice** accessible for everyone. It includes a planning template, mathematical practice look fors, "I can…" statements for students, and question starters, etc.

"Tools for Educators." Los Altos, CA: Inside Mathematics. Accessed at: www.insidemathematics.org/index.php/tools-for-teachers.
This link includes problems of the month to support the first standard for mathematical practice that can be used by individual teachers, departments, grade levels, or entire schools. The problems are structured in order to be accessible for a variety of levels K-12.

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